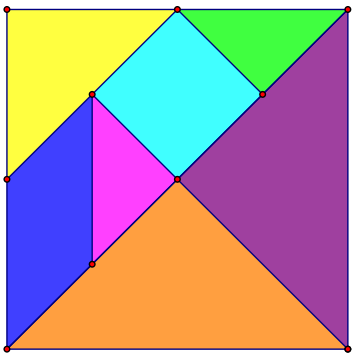
PLAYING WITH TANGRAMS

A tangram is an ancient Chinese puzzle. It is a square broken into seven parts (called 'tans') like so:

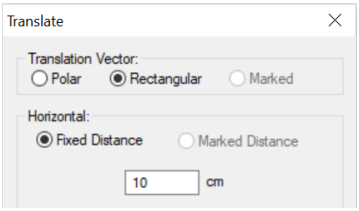


There are three different shapes made. What are they?

We are going to construct a tangram either using GSP or by hand. Your tangram is to measure 10cm by 10cm.

Here are the GSP instructions:

1. Place a point on the left-hand side near the bottom edge of the screen.



Select the point and translate it 10cm to the right. Go to Transform -> Translate, choose rectangular, then a horizontal distance of 10cm and a vertical distance of 0cm.

Click on Translate.

2. Click on line segment and connect the two points.
3. Using the arrow, double click on the right-hand point and highlight the line and the first point. Click on Transform and Rotate the line -90 degrees.
4. Repeat by double clicking the point at the top (make sure the vertical line and the point at the bottom are highlighted) and Rotate. Repeat until you have made a 10cm square.
5. Starting at the top left-hand point, label the vertices A, B, C, D. Note: You will need to double click the letters and change them.
6. Using the line segment tool, construct the diagonal BD.
7. Highlight the diagonal and construct the midpoint. This is point E.
8. Construct line segment CE.
9. We need the midpoints of DE and BE. To do this we need to draw in line segments connected these points. See if you can draw in the line segments and construct the midpoints. The midpoint of DE is F. The midpoint of BE is G.
10. Construct the midpoints of sides AB and AD. The midpoint of AD is H. The midpoint of AB is I.
11. Use the line segment tool to connect H and I.
12. Lastly, construct the midpoint of HI. This is point J.
13. Using the line segment tool, connect the following: JF, JE and IG.

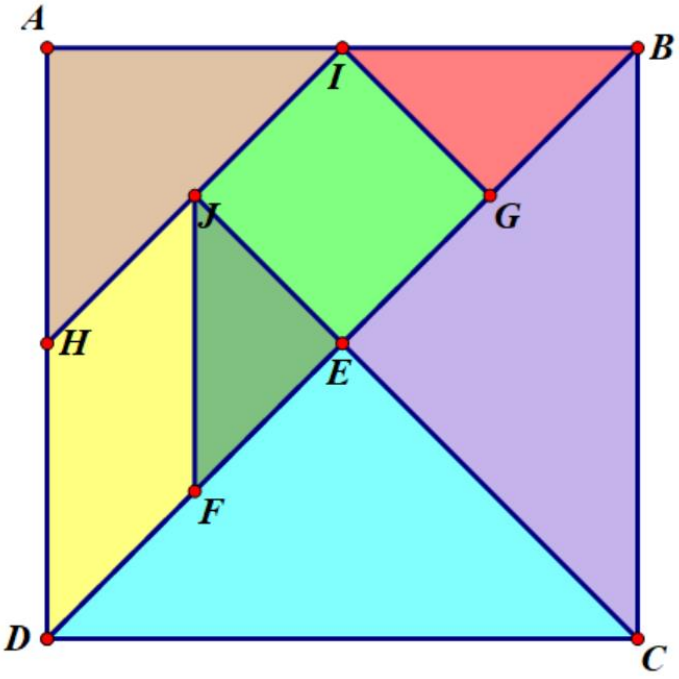
14. Now we are going to colour the interiors. Begin with triangle BEC. Highlight the three vertices, Construct, Triangle Interior.

15. Continue with all other shapes. Colour them different colours.

16. Save your gsp file in your Maths drive.

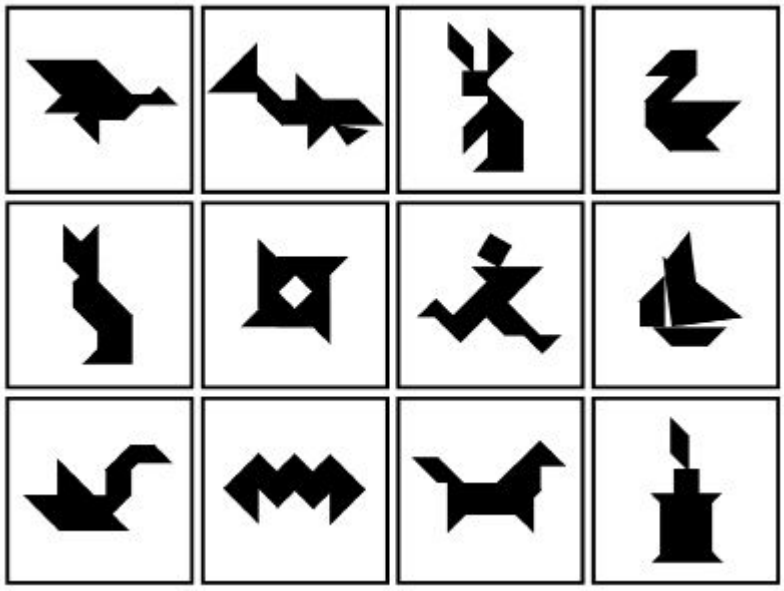
17. Print out your tangram. Make sure you print preview first and select Fit to Page. You may wish to select and hide each of the letters on the vertices.

18. Once you have printed out your tangram, cut it up into the seven tans.



MAKING SHAPES

The tangram was a puzzle in ancient China but was also very popular in Napoleon's court. The puzzle is to use the tans to make a particular shape, such as these below:



How many can you make?

Decide which one you like best and glue your tangram pieces into your workbook as a picture.

CALCULATING AREA

Now we are going to look at the areas of the different shapes.

What is the area of the total square?

Looking at the two larger triangles (DEC and BEC) what is a quick way to calculate their areas?

Now, let's calculate the area of triangle AHI. The area of a triangle is found using the formula:

$$A = \frac{1}{2}bh$$

where b is the base and h is the perpendicular height.

Remember the side length of the square is 10cm. What are the lengths AH and AI?

Calculate the area of the triangle.

What type of triangle is this? Give your answer in terms of the side lengths and the angle type.

Can you see a relationship between the small triangles JEF and BGI and the medium triangle AHI?

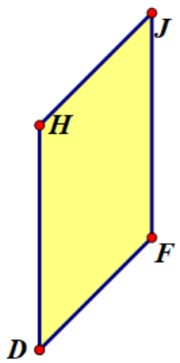
How can you use this to find the areas of the small triangles?

Use a similar trick to find the area of the square JIGE.

This leaves one shape remaining, the parallelogram. We are going to calculate the area two ways.

Firstly, the area of a parallelogram is found using the formula $A = bh$ where b is the base and h is the perpendicular height.

On the diagram below draw the perpendicular height in.



What is the length of this line?

What is the length of HD?

Calculate the area of the parallelogram

The other way we can calculate the area of the parallelogram is to total all the other areas and subtract them from the total area of the big square:

Complete:

Area of triangle BEC =

Area of triangle DEC =

Area of triangle AHI =

Area of triangle JEF =

Area of triangle BGI =

Area of square JIGE =

TOTAL =

Therefore, area of parallelogram =

Did you get the same answer?

There is another way to calculate the area of the parallelogram using proportions. What is it?

FRACTIONS AND PERCENTAGES

We used proportions to calculate the area of the two large triangles.

What fraction of the large square is triangle BEC?

What is this as a percentage?

What about the medium triangle AHI?

Now consider the small triangle JEF. What fraction of the large square is it?

Complete the table:

Shape	**Name**	**Fraction of ABCD**	**Percentage of ABCD**
Large Triangle	BEC		
Large Triangle	DEC		
Medium Triangle	AHI		
Small Triangle	JEF		
Small Triangle	BGI		
Small Square	JIGE		
Parallelogram	JFDH		
TOTALS			

HOW MANY COMBINATIONS? (EXTENSION)

Not surprisingly there are hundreds (maybe thousands) of different pictures that can be made from the seven tans.

Suppose we just lined the tans up in order like above, with all the shapes in the orientation shown, how many different combinations are there? We are going to consider the two large triangles as different triangles and the two medium triangles as different triangles.

Now let's consider the different orientations. We are only going to consider reflections and rotations about 90 degrees.

The square only has one orientation. It is the same when rotated and reflected.

Consider the parallelogram. Draw the number of possible orientations below.

How many are there?

So, if the parallelogram may rotate or reflect then we need to multiply our total number of combinations by the number of different orientations.

Total number of different combinations =

Now, let's look at the medium triangle. Draw the number of different orientations possible.

How many are there?

Total number of different combinations if the parallelogram and medium triangle are free to move

=

This will be the same for the other four triangles. How many different combinations are possible if orientations are allowed?

How many different 'pictures' do you think would be possible if the pieces are allowed anywhere?